KALMAN FILTER ESTIMATES OF THE NAVSTAR SATELLITE CLOCK PARAMETERS

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ABSTRACT

As is now well known, the Navstar/Global Positioning System is being used for precise time transfer between widely separated locations on earth. A part of this service is to provide users with information regarding the clocks in the GPS satellites. This data comes from the Kalman filter estimates in the Master Control Station at Vandenberg. This paper presents the results of an analysis of the Kalman filter data and illustrates the day-to-day characteristics of the satellite clock parameters that are provided to the time transfer user community. The monitor station tracking data also has been investigated and this paper illustrates the characteristics of the data acquired at the four monitor stations located at Vandenberg, Guam, Hawaii and Alaska. The paper shows that there are systematic and random variations in the Kalman filter estimates of the clock parameters, as well as in the tracking data. The relationship between the tracking data and the Kalman filter estimated parameters is discussed in this paper.

INTRODUCTION

Before going into the body of this paper, a brief overview of how GPS works is appropriate. There are now five fully operating satellites: Navstar 3, 4, 5, 6, and 8 - plus Navstar 1, which is operating with a quartz oscillator. Navstar 3, 4, and 8 are operating with rubidium clocks and Navstar 5 and 6 employ cesium beam frequency standards. These satellites are tracked by the four monitor stations located at Vandenberg in California, and at Guam, Hawaii, and Alaska. These stations track the satellites whenever they are visible and transmit the tracking data to the Master Control Station at Vandenberg. Here the data are processed and the ephemeris and clock parameters for each satellite are continuously updated. Actually, the update occurs every 15 minutes, 96 times per day. The upload station is also located at Vandenberg. At least once per day a revised estimate of the satellite's ephemeris and clock parameters is uploaded to the satellite which, in turn, transmits this data down on its navigating signal. In this way, GPS navigation users know the satellite's precise location in space, as well as the offset of the satellite clock from "GPS time." In general, GPS time is defined as the time of a selected monitor station clock, and usually the Vandenberg monitor station is selected for this purpose.

The ephemeris and clock parameters are updated in a complex Kalman filter. As of last August, Navstar 3, 4, 5, and 6 have their parameters estimated in a common partition, with a total of 57 parameters being solved for simultaneously. For each satellite there are 11 parameters (six ephemeris, two solar radiation pressure, and three clock states). There are two clock states for the three non-master monitor stations, a troposphere parameter for all four monitor stations, and three pole wander states.

A detailed study has been made of the day-to-day variations in the Kalman filter estimates of the satellite clock states, as well as in the satellite-to-monitor-station measurements. This results in a large mass of data. To limit the data to a manageable amount, this paper will discuss only one satellite, Navstar 3. The data covers the last two weeks of August 1983. Navstar 3 data was selected because it best illustrates the systematic behavior of the Kalman filter.

SATELLITE CLOCK PARAMETERS

The Kalman filter estimates three satellite clock parameters: phase, frequency, and frequency rate. All are estimated with reference to GPS time and, for the data presented here, the reference was the Vandenberg monitor station clock. Frequency rate is also called the "aging" parameter and is a parameter more peculiar to rubidium clocks than to cesium clocks.

A word about units may be of value. At the Master Control Station, the clock parameters are estimated in units of nanoseconds, nanoseconds per second, and nanoseconds per second squared for phase, frequency, and frequency rate, respectively. For the purpose of this paper, it was more meaningful to use units that relate closely to the navigation problem; instead of nanoseconds, meters are used and instead of seconds, days are used. Thus, the units are meters, meters per day, and meters per day squared.

A difficulty presented by the data is that the phase and frequency estimates can be very large values. In order to be able to "see" the data, the phase estimates are least-squares fitted to a straight line and only the residuals to this straight line fit are analyzed and plotted. Thus, the average phase and frequency are removed from the phase estimates. Likewise, the average frequency is removed from the Kalman filter frequency estimates. No adjustment is made to the Kalman filter estimates of frequency rate.

USER RANGE ERROR

A yardstick often used by the Navstar/GPS Program Office is "user range error" (URE). This consists of two components, ephemeris and clock. Let us concern ourselves only with the clock component. After the satellite is uploaded with the latest Kalman filter estimates of the three clock parameters, the filter continues to update its estimate of the satellite clock parameters. However, users must rely on the three parameters that were uploaded earlier in time. The new, revised estimate of the satellite clock phase is not available to the users until a new upload is sent up to the satellite. The difference between the current estimate of clock phase and the clock phase that users obtain from the satellite's navigation data message is the clock component of the user range error. It should be noted that the users depend heavily on the Kalman filter estimate of satellite clock frequency at the time of upload to determine the phase offset of the clock for the time they are navigating. On the other hand, the Kalman filter estimate of satellite clock phase does not have this dependability. This difference between the real-time Master Control Station determination of satellite clock phase and how the user obtains clock phase suggests the following as a reasonable measure of the difference in the data available at the two situations.

The basic question is the consistency between the Kalman filter estimates of phase and frequency. These parameters are the integral and differential of each other. A way to evaluate the performance of the Kalman filter at the Vandenberg Master Control Station is to numerically integrate the estimate of frequency, thereby creating a phase estimate based on the frequency estimate (this is akin to what the user must contend with). By subtracting this frequency-derived phase estimate from the actual real-time Kalman filter estimate of phase, a measure of the internal consistency of the Kalman filter is obtained. Several plots of this measure are given in this paper and will be related to the Navstar/GPS measurements obtained at the monitor stations.

MEASUREMENT RESIDUALS

The inputs to the Kalman filter are the smoothed measurements of pseudo-range and delta pseudo-range, which are obtained once every 15 minutes from every monitor station that can observe the satellite. These times are referred to as K-points. It should be noted that, with the current configuration of satellites and monitor stations, as many as 24 sets of measurements can, in principle, be made at each K-point. In practice, geometrical limitation limits the maximum to some lesser number; but, in any event, a great deal of data are fed into the filter. The delta pseudo-range data are the measured changes in the pseudo-range measurement within a 15-minute interval. It has been shown that the delta measurements play a very small role in the Kalman filtering process. For all intents and purposes, the filter operates almost entirely on the pseudo-range measurements.

The Kalman filter actually uses, as inputs, the so-called measurement residuals. The Master Control Station computes the expected value of pseudo-range, based on its latest estimate of the values of all the states being estimated. The residual is the difference between the actual pseudo-range measurement and the predicted value. Note that, if the system states were known perfectly, these two pseudo-ranges would be the same. Thus, the pseudo-range measurement residual is a measure of one or more of the states not being correctly estimated. At the present time, the tuning of the Kalman filter is such as to place most of the "blame" on the satellite clock parameters. The filter tends to adjust the satellite clock parameters more often than it does other system parameters, such as ephemeris and monitor station clocks.

In attempting to observe a correlation between the measurements and the satellite clock parameters, a simple device has been adopted to condense the measurement residual data. At each K-point all the measurement residuals from a particular satellite to whichever monitor stations can observe the satellite are added to form a composite total measurement residual. (The same convenience might be obtained by plotting all the measurement data on the same page.)

RESULTS

Analysis data have been collected for the period from 15 August 1983 through 28 August 1983. The Kalman filter estimates of the phase, frequency, and frequency rate of the rubidium clock on Navstar 3, relative to the Vandenberg monitor station clock, are shown on Figures 1, 2, and 3, respectively. In the case of phase, the plot is the residual from a least-squares fit to a straight line. The average has been removed from the frequency plot.

An examination of these figures reveals two distinct characteristics. For all three parameters there is a clearly repetitive day-to-day structure in their estimates. The second characteristic is the obvious correlation in the estimates of phase, frequency, and frequency rate. This flies in the face of mathematical reality in that one would not expect direct correlation between a quantity and its time differential (or time integral). The inexorable conclusion is that this repetitive structure and correlation between parameters is a reflection of the operation of the Kalman filter and is not a true day-to-day picture of satellite clock performance.

The data in Figure 4 show the integral of the Kalman filter frequency estimate and are, therefore, a phase measurement based on frequency. The initial value was set to the initial actual phase estimate shown in Figure 1. This integral is obviously smoother than the direct Kalman

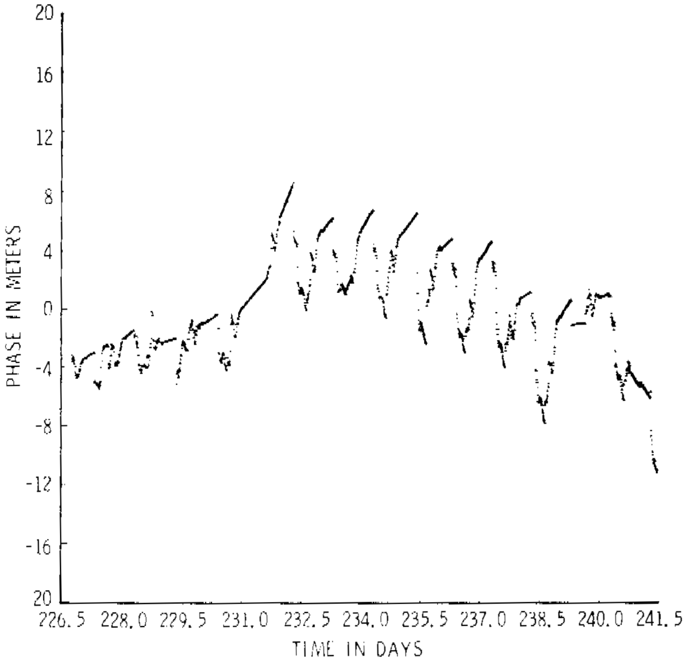

Figure 1. Kalman Clock Data, Navstar 3, Phase

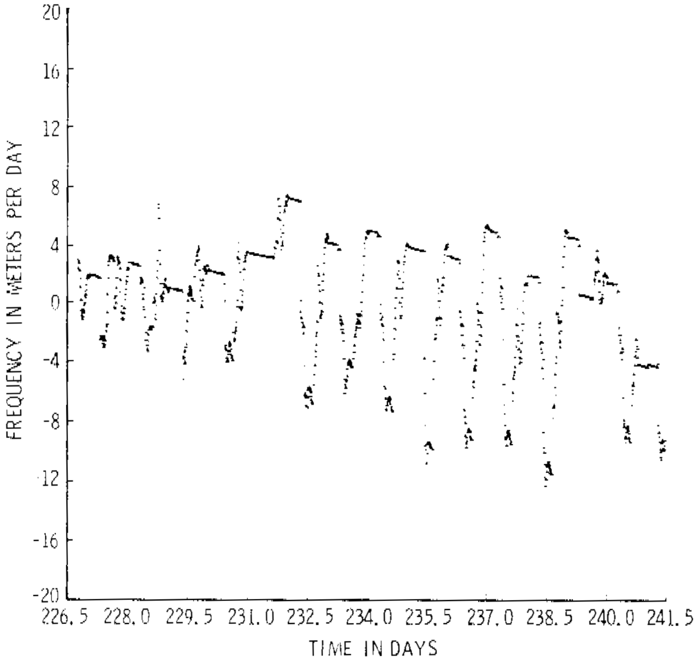

Figure 2. Kalman Clock Data, Frequency

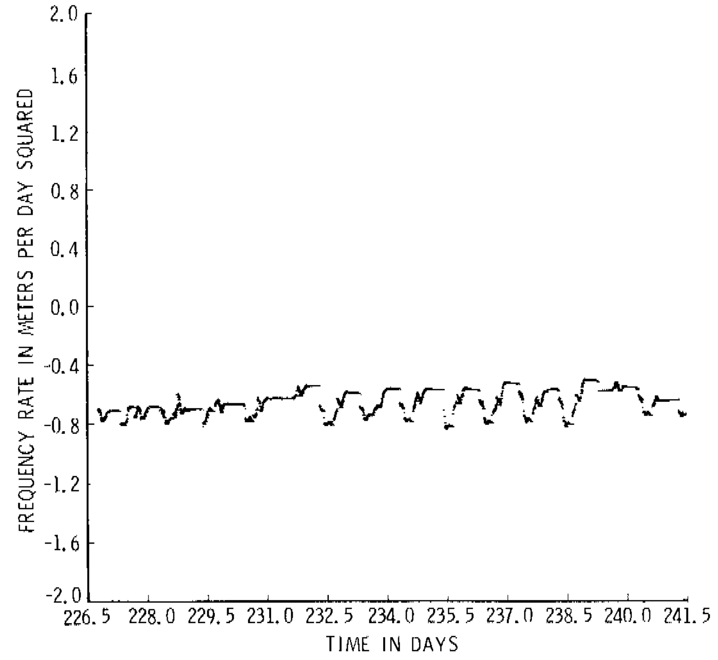

Figure 3. Kalman Clock Data, Frequency Rate

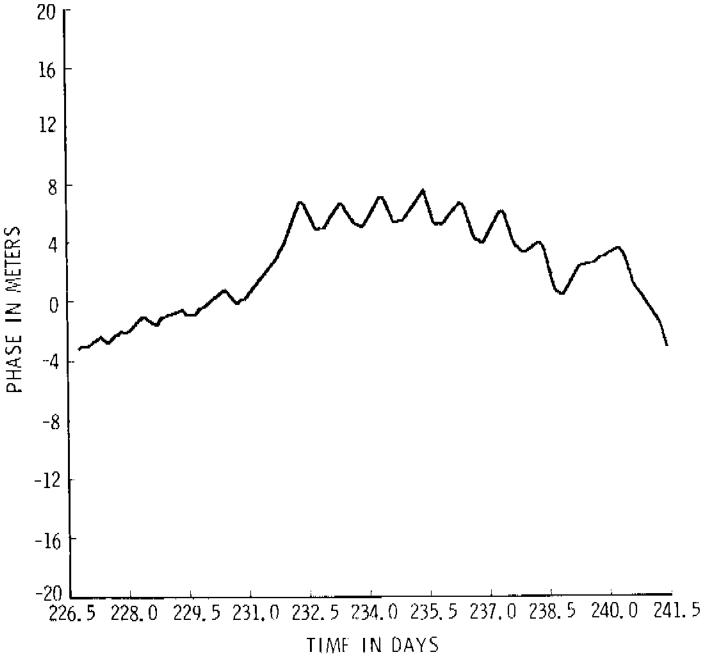

Figure 4. Kalman Clock Data, Frequency Integral

filter estimates shown in Figures 1 through 3. Yet, note the distinctive saw-tooth pattern over most of the two weeks. The difference between the actual real-time estimate of phase (Figure 1) and the phase obtained by integrating the frequency estimate (Figure 4) is shown in Figure 5. This quantity has been discussed previously and is akin to "user range error." It will be shown that this measure of satellite clock performance is related to the pseudo-range measurement residuals. Let us refer to the plot on Figure 5 as phase difference. The value of this representation is that it shows the day-to-day activity of the Kalman filter, whereas the long term (two weeks) variation in clock behavior is removed.

A part of the data on Figure 5 has been expanded to better observe the day-to-day response of the Kalman filter estimate of clock phase to the pseudo-range residuals. The data for five days, 22 August through 26 August 1983, are shown in Figures 6 through 10. A comparison of these five plots clearly shows the repetitive nature of the Kalman filter from one day to the next.

The pseudo-range measurement residuals at the four monitor stations are shown in Figures 11 through 14 for Vandenberg, Guam, Hawaii, and Alaska, respectively, for 22 August 1983. This corresponds in time to the phase difference Kalman filter estimates in Figure 6. A comparison of Figure 6 with Figures 11 through 14 reveals how the Kalman filter reacts to the inputs to the filter.

To better illustrate the relationship between the Kalman filter estimates of satellite clock phase and the measurement residuals, the residual data in Figures 11 through 14 have been combined. This is done simply by adding up the measurement residuals at the four monitor stations for each 15-minute K-point. The resulting total measurement residuals are shown in Figure 15. An examination of Figures 6 and 15 clearly shows how the Kalman filter estimate of satellite clock phase responded to the measurements on 22 August 1983. The total measurement residuals have also been obtained for the next four days, August 23 through 26. These total residuals are shown in Figures 16 through 19. Comparing Figures 7 and 16, 8 and 17, 9 and 18, and 10 and 19, gives a clear illustration of the response of the Kalman filter to the measurements for these four days. In addition, an examination of Figures 15 through 19 reveals the highly repetitive characteristics of the pseudo-range measurements from one day to the next. This, of course, accounts for the day-to-day repetition of the Kalman filter estimate of the satellite clock parameters.

DISCUSSION

With satellites 10,900 nmi above the earth and monitor stations located at intervals of several thousand miles across the Pacific, it should come as no surprise that some unknown bias lurks in the system. Some systematic effect is causing a day-to-day repeated pattern in the pseudo-range measurements taken at the monitor stations, an effect on the order of a few meters. Overall, the effect on the GPS user's navigation accuracy is quite acceptable. Nevertheless, being repetitive, it should be possible to root out this effect and eliminate it. Hopefully, this will be accomplished for the operational configuration of 18 satellites, five monitor stations distributed worldwide, and three or four ground antennas that allow for the uploading of each satellite three times per day.

The Kalman filter overreacts to the pseudo-range measurements taken at the monitor stations. It simply does not filter enough. The Kalman filter is "tuned" to a mythical world where all errors in the system emanate from a Gaussian, random noise process. That is not the real world, in which systematic, unknown effects are present. The strong correlation between the Kalman filter estimates of phase, frequency, and frequency rate (parameters that are differentials and integrals of each other) is a clear-cut indication of mis-tuning of the filter. The day-to-day repetition in these estimates, together with their overreaction to the measurement residuals, further supports this contention.

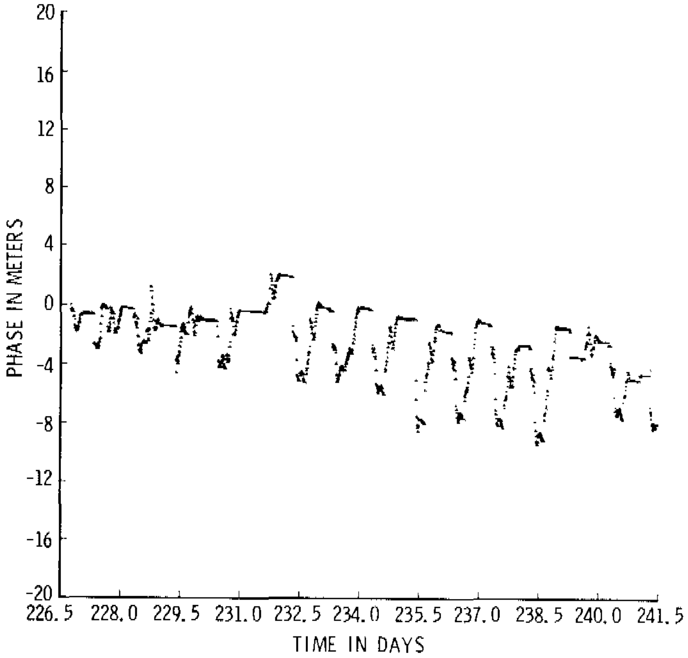

Figure 5. Kalman Clock Data, Phase Difference

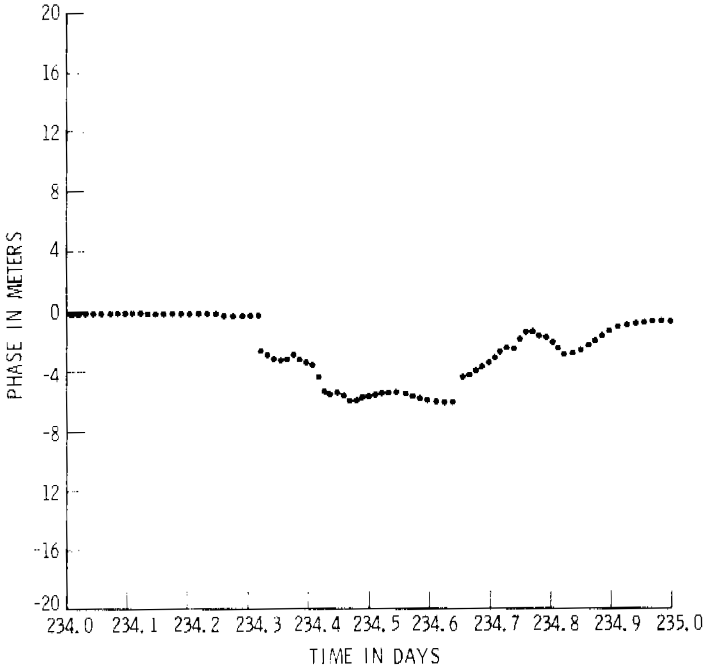

Figure 6. Kalman Clock Data, 22 August 1983

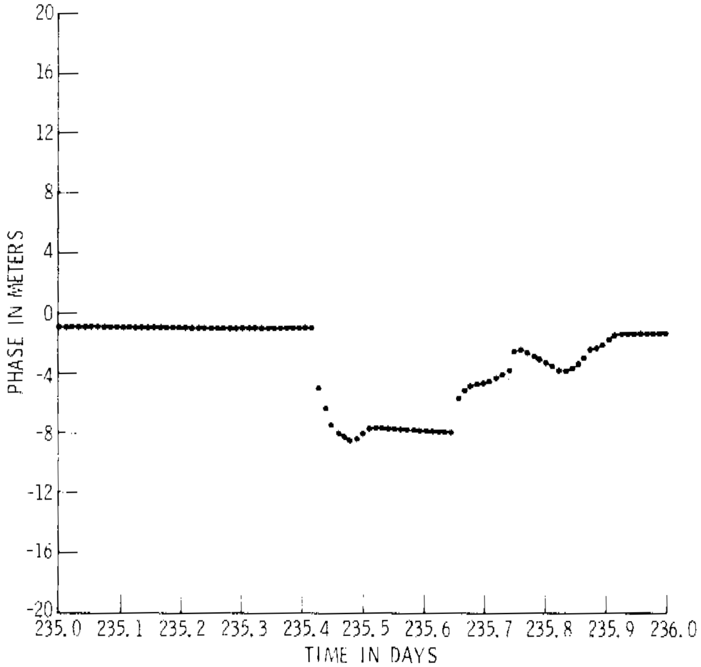

Figure 7. Kalman Clock Data, 23 August 1983

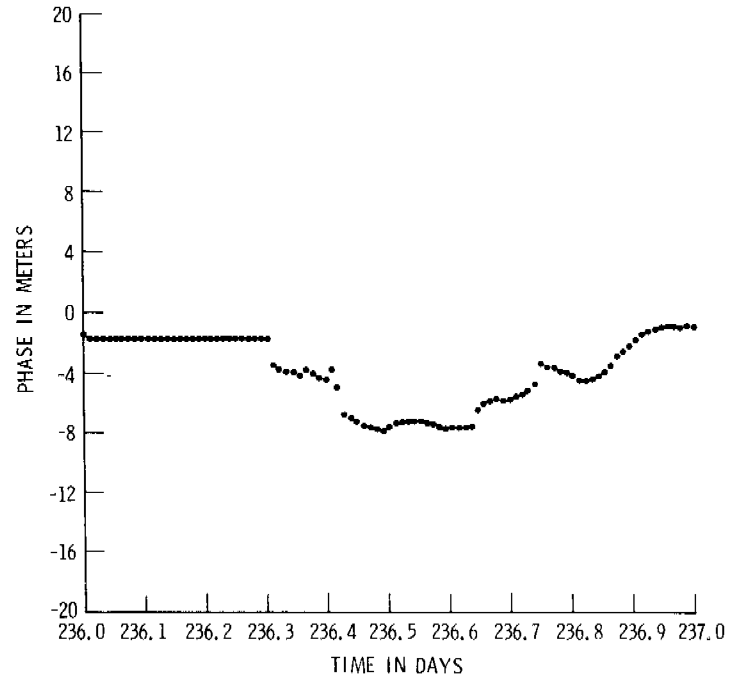

Figure 8. Kalman Clock Data, 24 August 1983

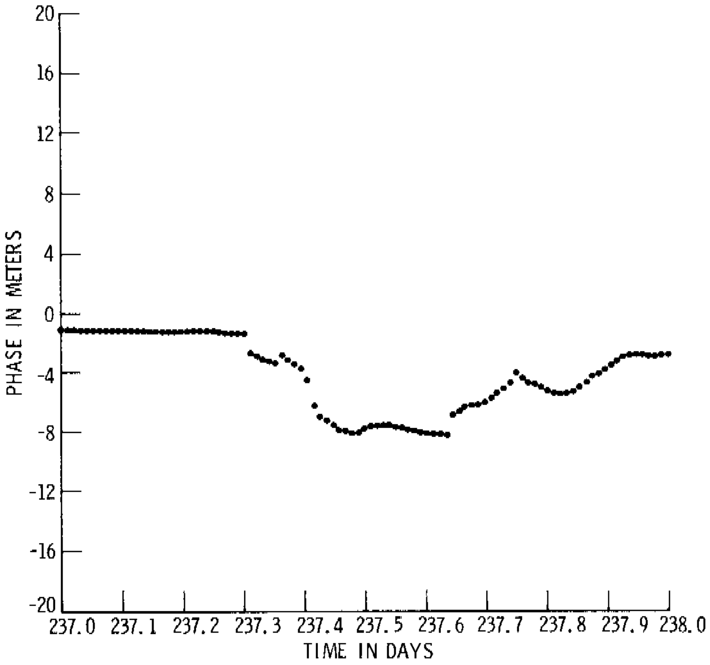

Figure 9. Kalman Clock Data, 25 August 1983

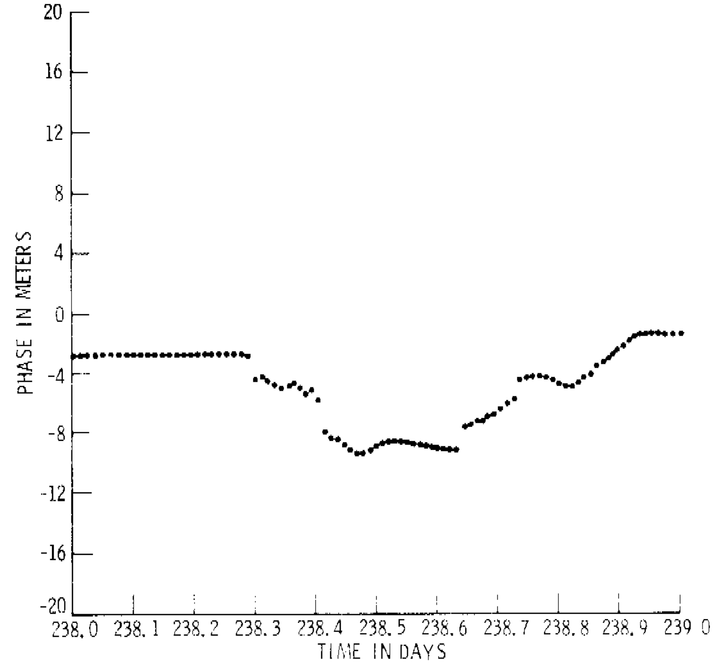

Figure 10. Kalman Clock Data, 26 August 1983

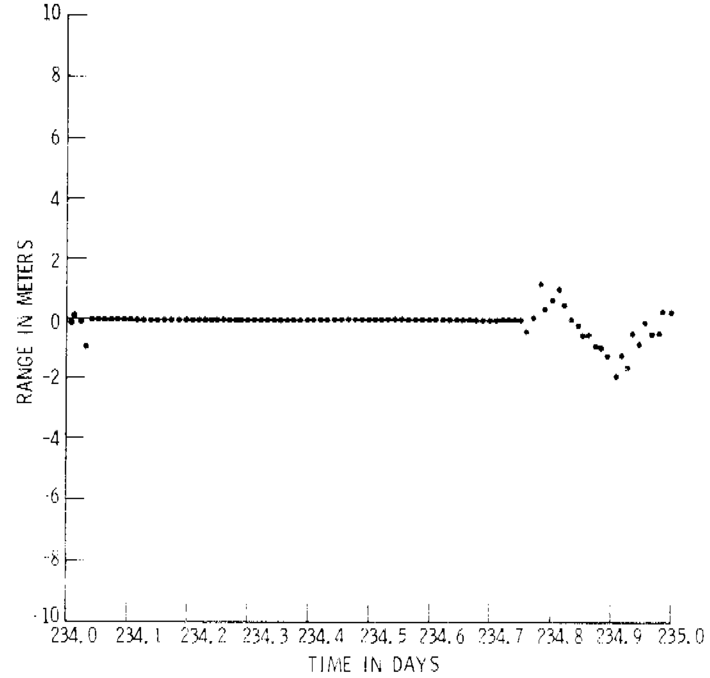

Figure 11. Kalman Residual Data, Vandenberg

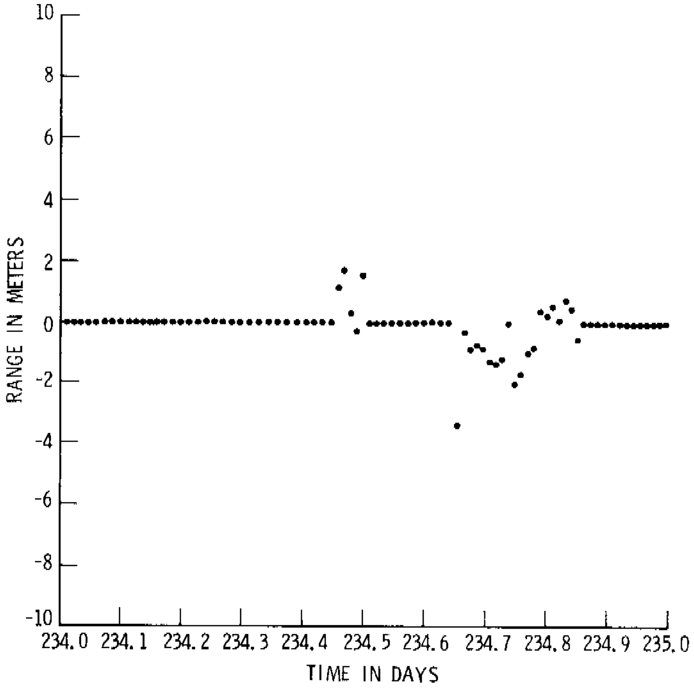

Figure 12. Kalman Residual Data, Guam

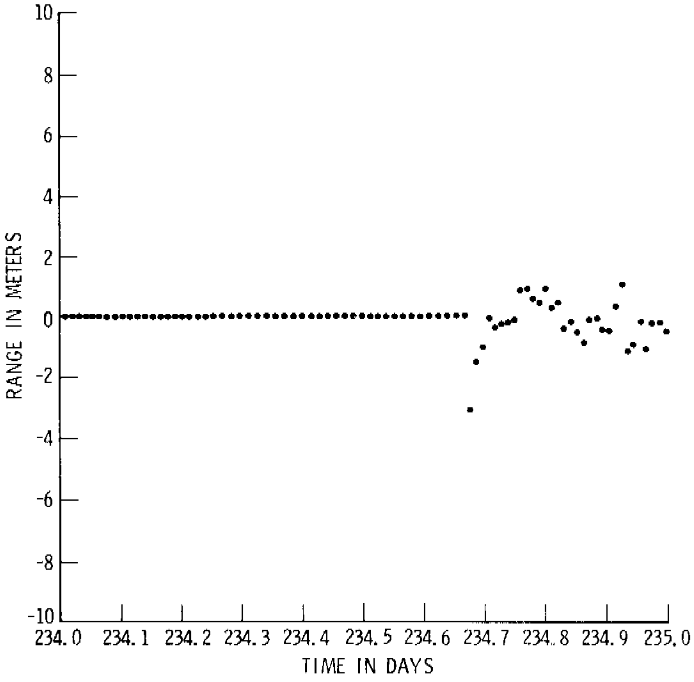

Figure 13. Kalman Residual Data, Hawaii

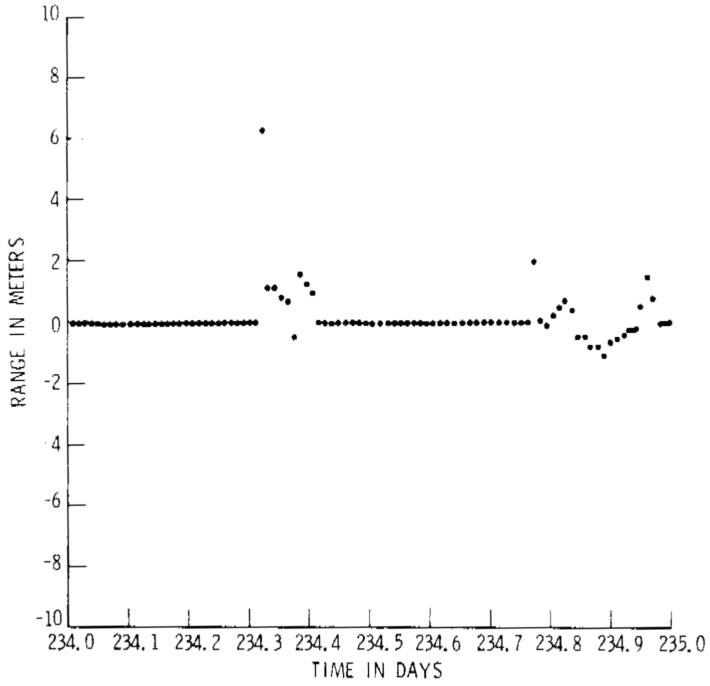

Figure 14. Kalman Residual Data, Alaska

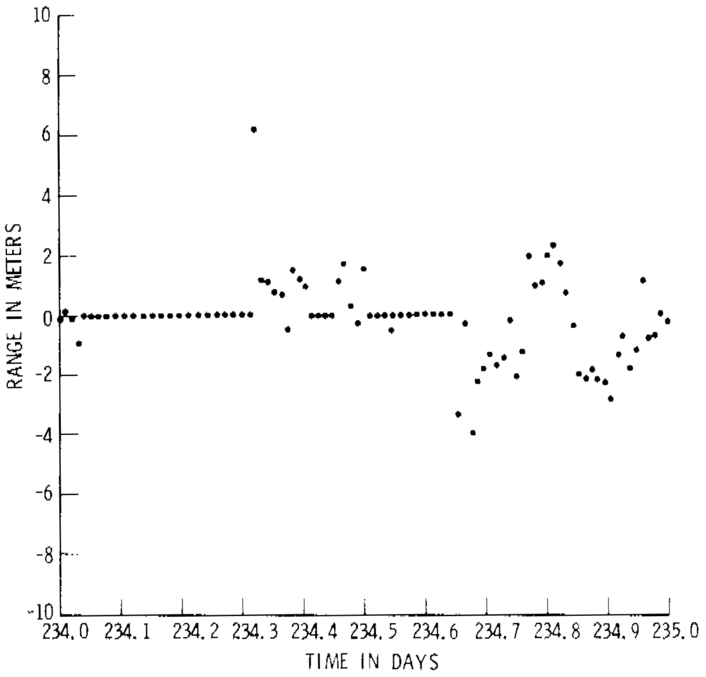

Figure 15. Kalman Residual Data, Total, 22 August 1983

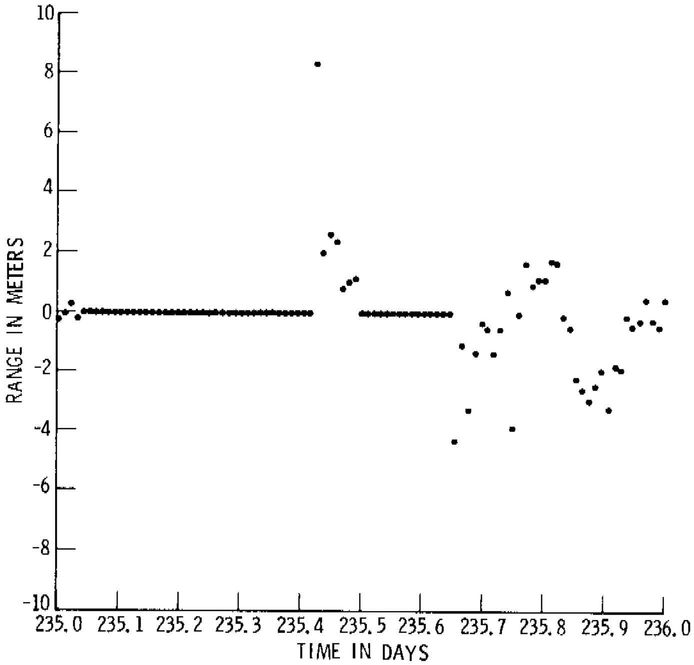

Figure 16. Kalman Residual Data, Total, 23 August 1983

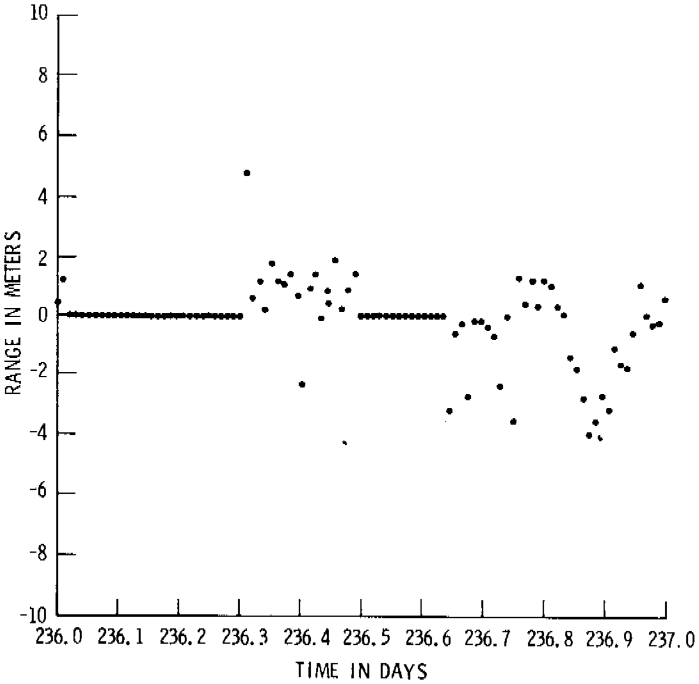

Figure 17. Kalman Residual Data, Total, 24 August 1983

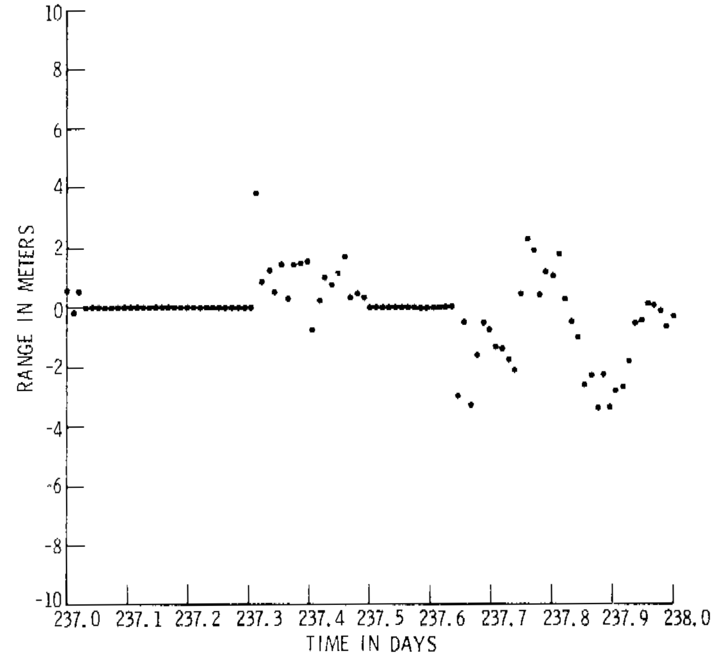

Figure 18. Kalman Residual Data, Total, 25 August 1983

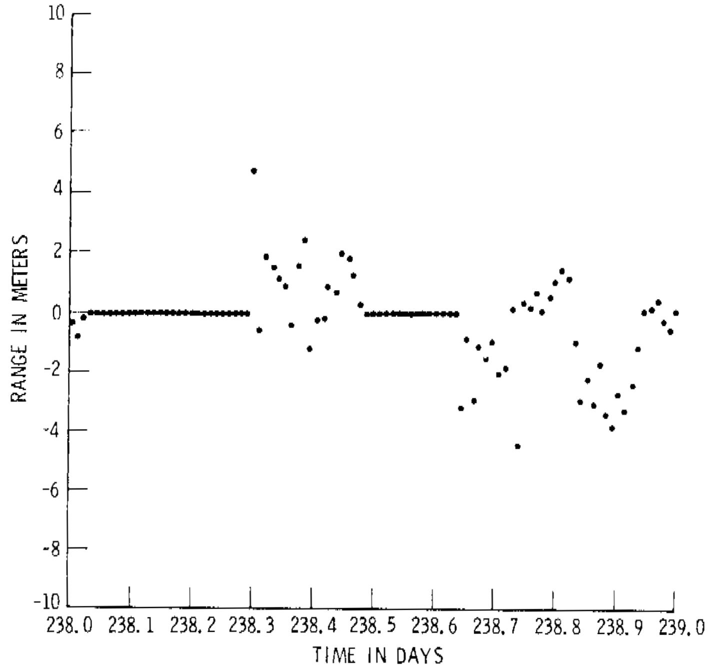

Figure 19. Kalman Residual Data, Total, 26 August 1983

QUESTIONS AND ANSWERS

MR. CARMAN:

I would like to ask one quick question. Are those epoch or current state plots?

MR. JORGENSEN:

These are current time.

MR. CARMAN:

I would like to point out for some that don't maybe know too much about G.P.S., that we are processing data for short periods each day, so that you would expect that you would get periodic phenomena with one-day periods. So the fact that you saw structure in the long term plots that had one-day periods, you could expect that.

In fact, between the time you process data during one day and the next day, you are propagating whatever states that have estimated for fifteen, maybe twenty, hours.

MR. JORGENSEN:

Right.

MR. CARMAN:

So that any miss-estimation you had in the ephemeris, even if you had perfect clock estimates, the random behavior in the clock is going to do something different than you are systematically trying to predict. So you would expect at the time of processing data the following day, you are going to have residuals with which to contend. I don't think those residuals that were shown were that bad.

MR. JORGENSEN:

That's right. The satellites are limited to fourteen-thousand miles away; and in the case of Alaska we should keep in mind that during one pass, the satellite is over there and in a second pass of satellite over there, so we are talking about enormous geometry; geometry that in totality is of the order of forty-thousand kilometers. We're talking about the worst case residual being eight meters. That point should be put into perspective.

MR. CARMAN:

The other point--because the ephemeris or satellite model behavior can be expected to be rather constant from one day to the next, when a residual appears in the order of five or six or eight meters, the first place the filter is going to try to put it is into the clock, specifically into the phase bacause it's the most observable state, directly correlating to range; one to one. So the immediate response of the filter is to adjust the phase to satisfy the measurement. So you might get a little over-reaction in the phase, but I think you have to let the filter settle a bit and that first residually doesn't mean very much. The fact that integrated frequency measurements, or frequency estimates, should be the same as your phase measurements, I don't understand that, because if that were really to be true, you wouldn't be estimating phase and frequency. You would only have to estimate the frequency state, integrate it and you would have the phase state.

So the filter is free, I think, within the mathematics to readjust its allocation between phase and frequency from one measurement to the next and there's no constraint that says that the integrated frequency estimate has to be the same as the phase estimate. I don't think it's an absurdity that it happens and I don't think it's a mathematical impossibility.

MR. ALLAN:

The analysis that we have done on separation of ephemeris that Dr. Weiss reported yesterday would very much support your conclusion that there are other systematics besides the clocks and those, in fact, are probably in the ephermeris. I think that strongly supports your conclusion.

DR. WINKLER:

One fact I think one has to accept is the measurement, and it's the time of arrival measurement, and you put the error into the clock because you don't want to do anything else. But my question is: does the Kalman filter--use, as a model, the oscillating eliptical elements?

MR. JORGENSEN:

Yes.

DR. WINKLER:

And it's quite clear that 12-hour orbit will have systematic perturbations with a period of only one day, half-day and multiples of that, and if his perturbations are not included in the filter, you would exactly expect the kind of performance that you get because we see the same thing. If we observe time measurements in distant places, we have again this up, down, up, down with a period of one day, and the amplitude is precisely the same as you have shown here.

MR. JORGENSEN:

All right. As Fran Varnum pointed out earlier, we use epoch states; that is to say, we use essentially the initial conditions, at say, the beginning of the two week period; however, in the operation of the whole process, we also use a two-week reference orbit developed by the Naval Surface Weapons Laboratory, which covers ahead from that two-week period which has the whole nine yards. It has all the physical things you can think about; all the gravity terms, solar radiation pressure, and what have you.

So it's really not quite oscillating elements. It's really the elements at zero time at the beginning of the two-week period.

DR. WEISS:

It's hard to believe that the time and frequency of the clocks themselves are correlated.

MR. JORGENSEN:

That's right. So the conclusion must be that what you saw regarding what the Kalman Filter is estimating has to do with the total estimation process, the combination of the ephemeris and clock estimation, and I think you must agree what must be is some really small systematic affect that, on a day-to-day basis, exists in the system.

DR. WEISS:

Well, my question is, how are the frequency and time of the clocks themselves measured, because it seems to me that what that implies is that there is a correlation in the measurement of those two quantities.

DR. JORGENSEN:

Well, the Kalman Filter estimates of the satellite clock parameters is it. That is what the world of users take the satellite clock to be doing; and, indeed, on a short term basis, there does appear to be some uncertainty in those estimates. But, on the other hand, as I pointed out, over the entire two-week period, the thing works.

You do have the proper relationship between frequency and phase, and if you notice also, the frequency rate plot appears to be consistent with the other two parameters over the two-week period. For all intents and purposes, the only thing we measure is the one-way ranging measurements from the satellite to the monitor stations.

DR. WEISS:

So you don't measure the frequency of the transmitted frequency at all? You just measure the time of arrival?

MR. JORGENSEN:

That's correct.

DR. WEISS:

Well, then you are not actually directly measureing the frequency of the clock.

DR. JORGENSEN:

That's correct. Well, I would like to clarify that. In the particular Kalman Filter we now use there is also a so-called Delta range measurement, which is within the fifteen minute period, the change in range. It turns out that if you look into the details of the thing, the way the filter works depends much more heavily on psuedo range measurements as opposed to the Delta psuedo range measurements.

MR. VANMELA:

Mr. Vanmela from Rockwell. To bring a little reality to the problem, we have a space craft up there right now, spacecraft II that was launched in July, and it just so happens that we have some Kalman Filter plots, nanoseconds per second and they have a very periodic change. Every day, it goes up and down, up and down, every day. So therefore, if everybody believed that, then the clock is really in trouble; but there seem to be other parameters involved, like the ephemeris data may be wrong or the Kalman Filter may be wrong, or slightly Q'd different. So that in the real world Rockwell has to look and Air Force has to look. We talked to each other, and the Air Force says the clock is jumping around too much, but it may be due to the application of the Kalman Filter. So that's what's actually happening. This was in the latter part of November.

DR. KLEPCZYNSKI:

Everybody seems to be accusing the ephemeris of the orbit. One thought that came to my mind here is that you are combining observations made at four different sites, which in my thoughts give you very poor geometry for orbit determination, in the first place. It seems that the inverse problem of--if there are any errors in the station coordinates at your four stations and they're systematically different, not of the same system, errors in this will find their way into the orbits or the satellites because you are making your observations, or combining your observations in the four different stations at different times and if each one is off in position and you find an error in your satellite ephemeris.

MR. JORGENSEN:

You are expressing the ovbious suspicion about something wrong with the coordinates of the Alaskan monitor station. On the other hand, I have been told that the station has been surveyed very accurately. So that's all I can say, as to the location of the monitor stations, don't forget the four locations were selected many years ago for the purpose of the Phase I program primarily to test out the system at Yuma, Arizona.

In the operational system we will have stations at totally different locations on earth, spread out around the entire globe, which of course will be better.